

# $\tau$ polarization asymmetries in $B^+ \rightarrow \tau^+ \nu_\tau \gamma$ and $B_c^+ \rightarrow \tau^+ \nu_\tau \gamma$

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## Abstract

We study the lepton polarization asymmetries in the radiative leptonic B decays of  $B^+ \rightarrow l^+ \nu_l \gamma$  and  $B_c^+ \rightarrow l^+ \nu_l \gamma$ . We concentrate on the transverse component of the  $\tau$  lepton asymmetries in the tau decay modes due to CP violation in theories beyond the standard model to search T violating effect.

# 1 Introduction

Both CP and T violations have been observed experimentally but so far they are only found in the neutral kaon system. It is known that CP violation implies T violation and vice versa because of the CPT theorem in the local quantum field theory with Lorentz invariance and the usual spin statistics. However, the origin of the CP or T violation is still unclear. In the standard model, CP violation arises from a unique physical phase in the Cabibbo-Kobayashi-Maskawa (CKM) [1] quark mixing matrix. To ensure the source of CP violation or T violation is this phase indeed, we need to consider new processes outside the kaon system.

In this report, we study the polarization asymmetries of the tau lepton in  $B^+ \rightarrow \tau^+ \nu_\tau \gamma$  and  $B_c^+ \rightarrow \tau^+ \nu_\tau \gamma$  decays and focus on the transverse parts ( $P_T$ ) of the asymmetries, which are related to the  $T$  odd triple correlation

$$P_T \propto \vec{s}_\tau \cdot (\vec{p}_\tau \times \vec{p}_\gamma), \quad (1)$$

where  $\vec{s}_\tau$  is the tau lepton spin vector and  $\vec{p}_i$  ( $i = \tau$  and  $\gamma$ ) are the momenta of the  $\tau$  and photon in the rest frame of the  $B$ -meson. As in the case of the radiative  $K_{\mu 2}^+$  decay [2], the CKM phase does not affect the transverse polarization in Eq. (1) and therefore, a non-zero value of  $P_T$  could be a signature of physics beyond the standard model. To illustrate our results, we will estimate the polarization in some typical non-standard CP violation theories, such as the left-right symmetric, three-Higgs doublet, and supersymmetric models. The tau transverse polarization for  $B^+ \rightarrow \tau^+ \nu_\tau \gamma$  could be measured in the two B factories at KEK and SLAC, while that for the  $B_c^+$  decay could be done at LHC where about  $2 \times 10^8$   $B_c$  mesons are estimated to be produced [3].

This report is organized as follows. In section 2, we give a general analysis for the lepton polarization asymmetries in the radiative leptonic B decays of interest. We also review the form factors for  $B_q \rightarrow \gamma$  ( $q = u, c$ ) transitions calculated directly in the entire physical range of momentum transfer within the light front framework [4, 5]. In section 3, we concentrate on the transverse part of the tau polarization asymmetries in  $B_q^+ \rightarrow \tau^+ \nu_\tau \gamma$  in various non-standard CP violation models. Our conclusions are summarized in section 4.

## 2 Lepton polarization asymmetries in $B_q^+ \rightarrow l^+ \nu_l \gamma$

For a general investigation of lepton polarization asymmetries in  $B_q^+ \rightarrow l^+ \nu_l \gamma$  including some new CP violating sources, we first carry out the most general four-fermion interactions given by

$$\begin{aligned} \mathcal{L} = & -\frac{G_F}{\sqrt{2}} V_{qb} \bar{q} \gamma^\alpha (1 - \gamma_5) b \bar{\nu} \gamma_\alpha (1 - \gamma_5) l + G_S \bar{q} b \bar{\nu} (1 + \gamma_5) l + G_P \bar{q} \gamma_5 b \bar{\nu} (1 + \gamma_5) l \\ & + G_V \bar{q} \gamma^\alpha b \bar{\nu} \gamma_\alpha (1 - \gamma_5) l + G_A \bar{q} \gamma^\alpha \gamma_5 b \bar{\nu} \gamma_\alpha (1 - \gamma_5) l + H.c. , \end{aligned} \quad (2)$$

where  $G_F$  is the Fermi constant,  $V_{qb}$  is the CKM mixing element,  $q = u$  and  $c$ , corresponding to  $B_u^+ \equiv B^+$  and  $B_c^+$  mesons,  $l = e, \mu$ , and  $\tau$ , and  $G_S, G_P, G_V$ , and  $G_A$  denote form factors for the scalar, pseudoscalar, vector, and axial vector interactions, arising from new physics, respectively.

In the standard model,  $G_i$  ( $i = S, P, V, A$ ) vanish and only the first term in Eq. (2) is present. The decay amplitudes for radiative leptonic B decays of  $B_q^+ \rightarrow l^+ \nu_l \gamma$  can be written into two parts

$$M(B_q^+ \rightarrow l^+ \nu_l \gamma) = M_{IB} + M_{SD} , \quad (3)$$

where  $M_{IB}$  and  $M_{SD}$  represent the “inner bremsstrahlung” (IB) and “structure-dependent” (SD) parts, and are given by

$$M_{IB} = ie \frac{G_F}{\sqrt{2}} V_{qb} f_{B_q} m_l \epsilon_\mu^* B^\mu , \quad (4)$$

$$M_{SD} = -i \frac{G_F}{\sqrt{2}} V_{qb} \epsilon_\alpha L_\beta H^{\alpha\beta} , \quad (5)$$

respectively, with

$$B^\mu = \bar{u}(p_\nu) (1 + \gamma_5) \left( \frac{p^\mu}{p \cdot q} - \frac{2p_l^\mu + \not{q} \gamma^\mu}{2p_l \cdot q} \right) v(p_l, s_l) , \quad (6)$$

$$L_\beta = \bar{u}(p_\nu) \gamma_\beta (1 - \gamma_5) v(p_l, s_l) , \quad (7)$$

$$H^{\alpha\beta} = e \frac{F_A^q}{M_{B_q}} (-g^{\alpha\beta} p \cdot q + p^\alpha q^\beta) + ie \frac{F_V^q}{M_{B_q}} \epsilon^{\alpha\beta\mu\nu} q_\mu p_\nu , \quad (8)$$

where  $\epsilon_\mu$  is the photon polarization vector,  $p, p_l, p_\nu$ , and  $q$  are the four momenta of  $B_q^+, l^+, \nu$  and  $\gamma$ , respectively,  $s_l$  is the polarization vector of the  $l^+$ ,  $f_{B_q}$  is the  $B_q$ -meson decay constant, and  $F_{A(V)}^q$  is the form factor of the vector (axial-vector) current. The factors  $f_{B_q}$  and  $F_{V,A}^q$  are defined as follows:

$$\langle 0 | \bar{q} \gamma^\mu \gamma_5 b | B_q(p) \rangle = -i f_{B_q} p^\mu , \quad (9)$$

$$\langle \gamma(q) | \bar{q} \gamma^\mu \gamma_5 b | B_q(p+q) \rangle = -e \frac{F_A^q}{M_{B_q}} [\epsilon^{*\mu}(p \cdot q) - (\epsilon^* \cdot p) q^\mu], \quad (10)$$

$$\langle \gamma(q) | \bar{q} \gamma^\mu b | B_q(p+q) \rangle = -ie \frac{F_V^q}{M_{B_q}} \epsilon^{\mu\alpha\beta\gamma} \epsilon_\alpha^* p_\beta q_\gamma. \quad (11)$$

Numerically,  $f_{B(B_c)} = 0.18$  (0.36)  $GeV$ , and  $F_A^q$  and  $F_V^q$  can be found in the Light-front quark model at the time-like momentum transfers in which the physical accessible kinematic region is  $0 \leq p^2 \leq p_{\max}^2$  with one loop level [4, 5].

For the IB amplitude, the charged  $B_q$ -meson emits leptons via the axial-vector current, and the photon is radiated from the external charged particles as shown in Figure 1. We can calculate the IB part by using the usual rules in QED and define it as the “trivial” part of the process  $B_q^+ \rightarrow l^+ \nu_l \gamma$ . We note that the amplitude,  $M_{IB}$ , is proportional to the ratio  $m_\ell/M_{B_q}$  and therefore it is helicity suppressed for the light charged lepton modes. The SD amplitude is governed by the vector and axial vector form factors shown in Figure 2, in which the photon is emitted from intermediate states. Gauge invariance leaves only two form factors,  $F_{V,A}^q$ , undetermined in the SD part. Since the transverse lepton polarization is deduced by the interference between IB and SD terms, it is clearly small in the processes of  $B_q^+ \rightarrow l^+ \nu_l \gamma$  with  $l = e$  and  $\mu$  as the smallness of  $M_{IB}$ . We shall focus on the decays of  $B_q^+ \rightarrow \tau^+ \nu_\tau \gamma$  and study the  $\tau$  polarization asymmetries. We remark that the discussions on the decay branching ratios of  $B_q^+ \rightarrow l^+ \nu_l \gamma$  in various models can be found in Refs. [4]-[11].

We now examine the probability of the process  $B_q^+ \rightarrow \tau^+ \nu_\tau \gamma$  as a function of the four momenta of the particles and the polarization vector  $s_\mu$  of the  $\tau$  lepton. We write the components of  $s_\mu$  in term of  $\vec{\eta}$ , the unit vector along the  $\tau$  lepton spin in its the rest frame, given by

$$s_0 = \frac{\vec{p}_\tau \cdot \vec{\eta}}{m_\tau}, \vec{s} = \vec{\eta} + \frac{s_0}{E_\tau + m_\tau} \vec{p}_\tau. \quad (12)$$

In the  $B_q^+$  rest frame, the partial decay rate is found to be

$$d\Gamma = \frac{1}{(2\pi)^3} \frac{1}{8M_{B_q}} |M|^2 dE_\gamma dE_\tau, \quad (13)$$

and

$$|M|^2 = A_0(x, y) + (A_L \vec{e}_L + A_N \vec{e}_N + A_T \vec{e}_T) \cdot \vec{\eta}, \quad (14)$$

where  $\vec{e}_i$  ( $i = L, N, T$ ) are the unit vectors along the longitudinal, normal and transverse components of the tau polarization, defined by

$$\vec{e}_L = \frac{\vec{p}_\tau}{|\vec{p}_\tau|},$$

$$\begin{aligned}
\vec{e}_N &= \frac{\vec{p}_\tau \times (\vec{q} \times \vec{p}_\tau)}{|\vec{p}_\tau \times (\vec{q} \times \vec{p}_\tau)|}, \\
\vec{e}_T &= \frac{\vec{q} \times \vec{p}_\tau}{|\vec{q} \times \vec{p}_\tau|},
\end{aligned} \tag{15}$$

respectively. The quantities of  $A_0$ ,  $A_L$ ,  $A_N$ ,  $A_T$  can be calculated directly and are given by

$$\begin{aligned}
A_0(x, y) &= \frac{1}{2}e^2 G_F^2 V_{qb}^2 (1-\lambda) \left\{ \frac{4m_\tau^2 |f_{B_q}|^2}{\lambda x^2} \left[ x^2 + 2(1-r_\tau) \left( 1-x - \frac{r_\tau}{\lambda} \right) \right] \right. \\
&\quad + M_{B_q}^4 x^2 \left[ |F_V^q + F_A^q|^2 \frac{\lambda^2}{1-\lambda} \left( 1-x - \frac{r_\tau}{\lambda} \right) + |F_V^q - F_A^q|^2 (y-\lambda) \right] \\
&\quad - 4M_{B_q} m_\tau^2 \left[ \text{Re}[f_{B_q}(F_V^q + F_A^q)^*] \left( 1-x - \frac{r_\tau}{\lambda} \right) \right. \\
&\quad \left. \left. - \text{Re}[f_{B_q}(F_V^q - F_A^q)^*] \frac{1-y+\lambda}{\lambda} \right] \right\}, \tag{16}
\end{aligned}$$

$$\begin{aligned}
A_L(x, y) &= -e^2 G_F^2 V_{qb}^2 \frac{(1-\lambda)}{2\lambda\sqrt{y^2-4r_\tau}} \left\{ \frac{4m_\tau^2 |f_{B_q}|^2}{\lambda x^2} [x(\lambda y - 2r_\tau)(x+y-2\lambda) \right. \\
&\quad \left. - (y^2 - 4r_\tau)(\lambda x + 2r_\tau - 2\lambda)] - M_{B_q}^4 \lambda x^2 \left[ |F_V^q + F_A^q|^2 \frac{\lambda}{1-\lambda} (\lambda y - 2r_\tau) \right. \right. \\
&\quad \left. \left. \left( 1-x - \frac{r_\tau}{\lambda} \right) + |F_V^q - F_A^q|^2 (y^2 - \lambda y - 2r_\tau) \right] \right. \\
&\quad - 4M_{B_q} m_\tau^2 \left[ \text{Re}\{f_{B_q}(F_V^q + F_A^q)^*\} \lambda \left( 1-x - \frac{r_\tau}{\lambda} \right) (2-2x-y) \right. \\
&\quad \left. \left. + \text{Re}\{f_{B_q}(F_V^q - F_A^q)^*\} ((1-y)(y-\lambda) + 2r_\tau - \lambda) \right] \right\}, \tag{17}
\end{aligned}$$

$$\begin{aligned}
A_N(x, y) &= e^2 G_F^2 V_{qb}^2 \frac{(1-\lambda)\sqrt{\lambda y - \lambda^2 - r_\tau}}{M_{B_q} \lambda \sqrt{y^2-4r_\tau}} \left\{ \frac{4m_\tau^3 |f_{B_q}|^2}{\lambda x} (x+y-2\lambda) \right. \\
&\quad - M_{B_q}^4 m_\tau \lambda x^2 \left[ |F_V^q + F_A^q|^2 \frac{\lambda}{1-\lambda} \left( 1-x - \frac{r_\tau}{\lambda} \right) + |F_V^q - F_A^q|^2 \right] \\
&\quad - 2M_{B_q}^3 m_\tau \left[ \text{Re}\{f_{B_q}(F_V^q + F_A^q)^*\} \left( \frac{(r_\tau - \lambda)(1-x-r_\tau)}{1-\lambda} + \lambda x(1-x) \right) \right. \\
&\quad \left. \left. - \text{Re}\{f_{B_q}(F_V^q - F_A^q)^*\} (y-2r_\tau) \right] \right\}, \tag{18}
\end{aligned}$$

$$\begin{aligned}
A_T(x, y) &= -2e^2 G_F^2 V_{qb}^2 M_{B_q}^2 m_\tau \frac{1-\lambda}{\lambda} \sqrt{\lambda y - \lambda^2 - r_\tau} \left\{ \text{Im}[f_{B_q}(F_V^q + F_A^q)^*] \frac{\lambda}{1-\lambda} \right. \\
&\quad \left. \times \left( 1-x - \frac{r_\tau}{\lambda} \right) + \text{Im}[f_{B_q}(F_V^q - F_A^q)^*] \right\}, \tag{19}
\end{aligned}$$

where  $\lambda = (x+y-1-r_\tau)/x$ ,  $r_\tau = m_\tau^2/M_{B_q}^2$ , and  $x = 2p \cdot q/p^2 = 2E_\gamma/M_{B_q}$  and  $y = 2p \cdot p_\tau/p^2 = 2E_\tau/M_{B_q}$  are normalized energies of the photon and  $\tau$ , respectively. If

we define the longitudinal, normal and transverse  $\tau$  polarization asymmetries by

$$P_i(x, y) = \frac{d\Gamma(\vec{e}_i) - d\Gamma(-\vec{e}_i)}{d\Gamma(\vec{e}_i) + d\Gamma(-\vec{e}_i)}, \quad (i = L, N, T), \quad (20)$$

we find that

$$P_i(x, y) = \frac{A_i(x, y)}{A_0(x, y)}, \quad (i = L, N, T). \quad (21)$$

From Eqs. (14), (15) and (20), it is easily seen that the asymmetries of  $P_L$  and  $P_N$  are even quantities under time-reversal transformation, while  $P_T$  is an odd one. Since we are interested in T or CP violation, we shall give a detail discussion only in the transverse part of the polarization asymmetries. Clearly, to have T-odd transverse  $\tau$  polarization of  $P_T$  in Eq. (21), at least one of the form factors,  $f_{B_q}$  and  $F_{A,V}^q$ , has to be complex. In the standard model, from Eqs. (19) and (21), we see that the longitudinal and normal  $\tau$  polarizations are non-vanishing but  $P_T = 0$  since  $f_{B_q}$  and  $F_{V,A}^q$  are all real at the tree level. This result is trivial since there is only one diagram which contributes to the process of  $B_q^+ \rightarrow \tau^+ \nu_\tau \gamma$  and there is no interference effect or CP violation. At the loop level, non-zero value of  $P_T$  can be induced by electromagnetic final state interactions and it is estimated to be  $O(10^{-3})$  similar to the kaon case [2]. To distinguish the real CP violating effects from the final state interactions at the level of  $10^{-3}$ , one may measure the difference between the tau polarization in  $B_q^+ \rightarrow \tau^+ \nu_\tau \gamma$  and  $B_q^- \rightarrow \tau^- \bar{\nu}_\tau \gamma$  since the final state interactions give the same  $P_T$  in both modes whereas CP violation yields a different sign for  $P_T$ . Beyond the level of  $10^{-3}$ , an observation of  $P_T$  in  $B_q^+ \rightarrow \tau^+ \nu_\tau \gamma$  will indicate T violation originated from new physics.

For  $B^+$  meson, in which  $q = u$ , using  $|V_{ub}| \simeq 3 \times 10^{-3}$ ,  $M_B = 5.2 \text{ GeV}$ ,  $m_\tau = 1.78 \text{ GeV}$ ,  $f_B = 0.18 \text{ GeV}$ ,  $\tau_{B^+} \simeq 1.62 \times 10^{-12} \text{ s}$  [12], and  $\omega = 0.57 \text{ GeV}$ , we find that the integrated branching ratio of  $B^+ \rightarrow \tau^+ \nu_\tau \gamma$  is given by<sup>a</sup>

$$Br(B^+ \rightarrow \tau^+ \nu_\tau \gamma) \simeq 5.8 \times 10^{-7}. \quad (22)$$

For the decay of  $B_c^+ \rightarrow \tau^+ \nu_\tau \gamma$ , with  $|V_{cb}| \simeq 4 \times 10^{-2}$ ,  $M_{B_c} = 6.4 \text{ GeV}$ ,  $f_{B_c} = 0.36 \text{ GeV}$ , and  $\tau_{B_c^+} \simeq 0.46 \times 10^{-12} \text{ s}$  [13, 14], we obtain [5]

$$Br(B_c^+ \rightarrow \tau^+ \nu_\tau \gamma) \simeq 1.1 \times 10^{-4}. \quad (23)$$

In Figure 3, we show the Dalitz plots of  $A_0$  for  $B_{u,c}^+ \rightarrow \tau^+ \nu_\tau \gamma$ . We remark that for the mode  $B_q^+ \rightarrow l^+ \nu_l \gamma$  with  $l = e$  or  $\mu$ , the transverse lepton polarization asymmetry vanishes

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<sup>a</sup>Our result here for  $B^+ \rightarrow \tau^+ \nu_\tau \gamma$  agrees with those for  $B^+ \rightarrow l^+ \nu_l \gamma$  ( $l = e, \mu$ ) shown in Ref. [4] after a correction is made. See the note of [19] in Ref. [5].

in the limit of  $m_l \rightarrow 0$  by notifying the lepton mass dependence in the expression of Eq. (19). In such case, the lepton in the decay is almost 100% longitudinal polarized. This is why we choose  $B_q^+ \rightarrow \tau^+ \nu_\tau \gamma$  to search  $P_T$  instead of using  $B_q^+ \rightarrow \mu^+ \nu_\mu \gamma$ .

In the non-standard models,  $P_T(B_q^+ \rightarrow \tau^+ \nu_\tau \gamma)$  can be non-zero if new physics contains some CP violating phases which lead to CP violating physical couplings. These shall be discussed in the next section. However, it is interesting to note that there is no contribution to the transverse  $\tau$  polarization if the interaction beyond the standard model has only left-handed vector current, because there is no relative phase between the amplitudes of  $M_{IB}$  and  $M_{SD}$ .

### 3 Transverse $\tau$ Lepton Polarization in Non-standard Theories

It is known that many theories beyond the the standard model provide new CP-violation phases outside the CKM mechanism. In these theories, sizable  $P_T$  could be induced. In this section, we will estimates the values of  $P_T$  in the left-right symmetric, three-Higgs-doublet, and supersymmetric models, respectively.

From the interactions in Eq. (2), similar to the discussion in section 2, we can write the amplitude of the decay  $B_q^+ \rightarrow \tau^+ \nu_\tau \gamma$  in terms of the IB and SD contributions. However, the factors  $f_{B_q}$  and  $F_{A,V}^q$  in Eqs. (4)-(8) need to be replaced as follows [2]:

$$\begin{aligned} f_{B_q} &\rightarrow f_{B_q}(1 + \Delta_P + \Delta_A), \\ F_A^q &\rightarrow F_A^q(1 + \Delta_A), \\ F_V^q &\rightarrow F_V^q(1 - \Delta_V), \end{aligned} \tag{24}$$

with

$$\Delta_{(P,A,V)} = \frac{\sqrt{2}}{G_F V_{qb}} \left( \frac{G_P M_{B_q}^2}{(m_b + m_q) m_\tau}, G_A, G_V \right). \tag{25}$$

In Eq. (24), the factors of  $\Delta_{(P,A,V)}$  could be complex numbers due to new physical phases arising from  $G_P, G_V, G_A$ , respectively. We rewrite  $P_T(x, y)$  in Eq. (20) as

$$P_T(x, y) = P_T^V(x, y) + P_T^A(x, y) \tag{26}$$

with

$$\begin{aligned} P_T^V(x, y) &= \sigma_V(x, y) [Im(\Delta_A + \Delta_V)], \\ P_T^A(x, y) &= [\sigma_V(x, y) - \sigma_A(x, y)] Im(\Delta_P), \end{aligned} \tag{27}$$

where

$$\begin{aligned}
\sigma_V(x, y) &= 2e^2 G_F^2 V_{qb}^2 M_{B_q}^2 m_\tau f_{B_q} F_V^q \frac{\sqrt{\lambda y - \lambda^2 - r_\tau}}{\rho_0(x, y)} \\
&\quad \times \left( \frac{-1 + \lambda}{\lambda} - \left( 1 - x - \frac{r_\tau}{\lambda} \right) \right), \\
\sigma_A(x, y) &= 2e^2 G_F^2 V_{qb}^2 M_{B_q}^2 m_\tau f_{B_q} F_A^q \frac{\sqrt{\lambda y - \lambda^2 - r_\tau}}{\rho_0(x, y)} \\
&\quad \times \left( \frac{-1 + \lambda}{\lambda} + \left( 1 - x - \frac{r_\tau}{\lambda} \right) \right). \tag{28}
\end{aligned}$$

It is clear that, in order to have a non-zero T violating  $P_T$  for  $\tau$ , the coupling constants  $G_{(P,A,V)}$  have to be present and furthermore at least one of them is complex to provide the CP violating phase. In Figures. 4 and 5, we display the Dalitz plots of  $\sigma_V(x, y)$  and  $\sigma_V(x, y) - \sigma_A(x, y)$  in  $B_{u,c}^+ \rightarrow \tau^+ \nu_\tau \gamma$ , respectively. From the figures, we see that they are all in the order of  $10^{-1}$  in most of the allowed parameter space.

### 3.1 Left-right symmetric models

We now study the T violating  $\tau$  polarization for decay  $B_q^+ \rightarrow \tau^+ \nu_\tau \gamma$  in models with  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  gauge symmetries. In these models, the Higgs multiplets break the symmetries down to  $U(1)_{em}$  [2] and the masses of fermions are generated from the Yukawa couplings. In addition, the mass eigenstates of gauge bosons  $W_{L,R}$  are related to weak eigenstates, given by

$$\begin{aligned}
W_1 &= \cos \xi W_L + \sin \xi W_R, \\
W_2 &= -\sin \xi W_L + \cos \xi W_R, \tag{29}
\end{aligned}$$

where  $\xi$  is the left-right mixing angle. Due to the  $W_R$  gauge boson, we have the new mixing matrix called the right-handed CKM (RCKM) matrix. In these models, the quark level four-fermion interaction is given by

$$\mathcal{L}_{RL} = -2\sqrt{2}G_F \left(\frac{g_R}{g_L}\right) K_{qb}^{R*} \xi \bar{b} \gamma_\mu P_R q \bar{\nu} \gamma^\mu P_L \nu, \tag{30}$$

which contributes  $B_q^+ \rightarrow \tau^+ \nu_\tau \gamma$ , where  $g_{L,R}$  are coupling constants for  $SU(2)_{L,R}$  and  $K_{qb}^{R*}$  is the RCKM matrix element. Here, we do not assume parity invariant and any special relation between the CKM and RCKM matrices. In general, the gauge coupling constants  $g_L$  and  $g_R$  are unequal to each other and they are free parameters. Compare  $\mathcal{L}$  in Eq. (2) with  $\mathcal{L}_{RL}$  in Eq. (30), we find that

$$G_A = G_V = -\frac{G_F}{\sqrt{2}} \left(\frac{g_R}{g_L}\right) K_{qb}^{R*} \xi. \tag{31}$$



Because we have the relations between  $\Delta_{A,V}$  and  $G_{A,V}$  in Eq. (25), we get

$$\Delta_A = \Delta_V = -\frac{K_{qb}^{R*} \xi}{V_{qb}} \left( \frac{g_R}{g_L} \right). \quad (32)$$

From the definition of the  $\tau$  transverse polarization, we obtain

$$P_T = 2\sigma_V \left( \frac{g_R \xi}{g_L} \right) \text{Im}(K_{qb}^{R*}). \quad (33)$$

In the typical left-right symmetric models shown in Ref. [15], one generally has that

$$\xi \frac{g_R}{g_L} < 4.0 \times 10^{-3}. \quad (34)$$

The limit in Eq. (34) clearly leads to a very small  $P_T$  even with a large value of  $\text{Im}(K_{qb}^{R*})$ . However, in a class of the specific models studied in Ref. [16], it is found that

$$\xi \frac{g_R}{g_L} < 3.3 \times 10^{-2} \quad (35)$$

for  $M_R > 549 \text{ GeV}$ . With the value in Eq. (35) and the assumption of  $2\sigma_V \text{Im}(K_{qb}^{R*}) \sim 1$ , we get that

$$P_T(B_q^+ \rightarrow \tau^+ \nu_\tau \gamma) < 3.3 \times 10^{-2}, \quad (36)$$

for both  $q = u$  and  $c$  modes. The bound in Eq. (36) can be larger if one uses a large value of  $\sigma_V$ . However, it is hard to be measured in B factories and LHC for the  $B^+$  and  $B_c^+$  decays, respectively. We remark that similar to the charged kaon case [2, 17], the transverse lepton polarizations for  $B_q^+ \rightarrow (\pi^0, D^0) l^+ \nu_l$  vanish, whereas that for  $B_q^+ \rightarrow D^* l^+ \nu_l$  are expected to be non-zero since both  $\gamma$  and  $D^*$  are vector particles while  $\pi^0$  and  $D^0$  are pseudoscalars.

### 3.2 Three-Higgs-Doublet Models

In the Standard Model (SM), there is no tree level flavor changing neutral current (FCNC) because the fermion masses and Higgs-fermion couplings can be simultaneously diagonalized. In a multi-Higgs doublet model (MHDM), the spontaneous  $CP$  violation (SCPV) arises from complex vacuum expectation values. In the Weinberg three-Higgs-doublet model (THMD) [18],  $CP$  phase occurs in the charged-Higgs-boson mixing and it contributes to FCNC at tree level. We shall concentrate on this phase and assume that the CKM matrix is real in this subsection.

With the consideration of the natural flavor conservation (NFC) [19], the general Yukawa interaction in the models is given by

$$\mathcal{L}_Y = \bar{Q}_{L_i} F_{ij}^D \phi_d D_{R_j} + \bar{Q}_{L_i} F_{ij}^U \tilde{\phi}_u U_{R_j} + \bar{L}_{L_i} F_{ij}^E \phi_e E_{R_j} + h.c. \quad (37)$$

with  $i, j = 1, 2, 3$ , where  $Q_{L_i}$ ,  $L_{L_i}$  are the left-handed quark and lepton doublets, while  $U_{R_j}$ ,  $D_{R_j}$  and  $E_{R_j}$  are right-handed singlets for up, down-type quarks and charged leptons,  $\phi_k$  ( $k = u, d, e$ ) are Higgs doublets with  $\tilde{\phi}_k = i\sigma_2\phi_k^*$ , and  $F_{ij}$  are the coupling constants, respectively. With fermion mass eigenstates, the Yukawa interaction of physical charged scalars is given by

$$\mathcal{L} = (2\sqrt{2}G_F)^{1/2} \sum_{i=1}^2 (\alpha_i \bar{U}_L K M_D D_R + \beta_i \bar{U}_R M_U K D_L + \gamma_i \bar{\nu}_L M_E E_R) H_i^+ + h.c., \quad (38)$$

where  $M_D$ ,  $M_U$ , and  $M_E$  are the diagonal mass matrices of down, up-type quarks and charged leptons, respectively,  $K$  is the mixing matrix for quark sector,  $H_i^+$  ( $i = 1, 2$ ) denote the two charged physical scalars and  $\alpha_i$ ,  $\beta_i$ ,  $\gamma_i$  are complex coupling constants which are related by

$$\frac{Im\alpha_1\beta_1^*}{Im\alpha_2\beta_2^*} = \frac{Im\alpha_1\gamma_1^*}{Im\alpha_2\gamma_2^*} = \frac{Im\beta_1\gamma_1^*}{Im\beta_2\gamma_2^*} = -1. \quad (39)$$

Comparing Eq. (38) with Eq. (2), we find that the pseudoscalar coupling constant  $G_P$  has the following form

$$G_P = \sqrt{2}G_F V_{qb} m_\tau \sum_{i=1}^2 \frac{\gamma_i}{m_i^2} (m_q \beta_i^* - m_b \alpha_i^*). \quad (40)$$

From Eq. (25), one has that

$$Im\Delta_P = \frac{M_{B_q}^2}{m_b + m_q} \sum_{i=1}^2 Im \frac{\gamma_i}{m_i^2} (m_q \beta_i^* - m_b \alpha_i^*). \quad (41)$$

To illustrate the polarization effect, we assume that  $m_{H_2} \gg m_{H_1} \equiv m_H$ . As shown in Refs. [20, 21], for  $m_H < 400 \text{ GeV}$ , the experimental limit on the inclusive process  $B \rightarrow X \tau \nu_\tau$  decay gives the strongest bound

$$\frac{|Im\alpha_1^*\gamma_1|}{m_H^2} < 0.23 \text{ (GeV)}^{-2}, \quad (42)$$

which leads to

$$|Im\Delta_P|_{u,c} < 5.8, 6.7 \quad (43)$$

where we have ignored the terms relating to  $m_q$  in Eq. (41). From Eq. (27), with  $|\sigma_V - \sigma_A| \sim 0.1$  and the limit in Eq. (43), the transverse  $\tau$  polarization asymmetries of  $B_{u,c}^+ \rightarrow \tau^+ \nu_\tau \gamma$  are found to be

$$P_T(B_{u,c}^+ \rightarrow \tau^+ \nu_\tau \gamma) < 0.58, 0.67. \quad (44)$$

Clearly, a measurement of  $P_T$  would also constrain  $Im\Delta_P$ . We remark that large  $P_T(B^+ \rightarrow M l^+ \nu_l)$  with  $M = \pi, D^{(*)}$  for  $l = \mu, \tau$  are also expected [22].

### 3.3 Supersymmetric Models without R-parity

It is known that, in general, SUSY theories would contain couplings with the violation of baryon or/and lepton numbers, that could induce the rapid proton decay. To avoid such couplings, one usually assigns R-parity, defined by  $R \equiv (-1)^{3B+L+2S}$  to each field, where  $B(L)$  and  $S$  stand for the baryon (lepton) number and spin, respectively. Thus, the R-parity can be used to distinguish the particle ( $R=+1$ ) from its superpartner ( $R=-1$ ). In this study, we will discuss the SUSY models without R-parity. To evade the stringent constraint from proton decay, we simply require that B violating couplings do not coexist with the L violating ones. With the violation of the R-parity and the lepton number, we write the superpotential as

$$W_{\mathcal{U}} = \frac{1}{2} \lambda_{ijk} L_i L_j E_k^c + \lambda'_{ijk} L_i Q_j D_k^c, \quad (45)$$

where the subscripts  $i$ ,  $j$  and  $k$  are the generation indices,  $L$  and  $E^c$  denote the chiral superfields of lepton doublets and singlets, and  $Q$  and  $D^c$  are the chiral superfields of quark doublets and down-type quark singlets, respectively. We note that the first two generation indices of  $\lambda_{ijk}$  are antisymmetric, *i.e.*,  $\lambda_{ijk} = -\lambda_{jik}$ . The corresponding Lagrangian is

$$\begin{aligned} \mathcal{L}_{\mathcal{U}} = & \frac{1}{2} \lambda_{ijk} [\bar{\nu}_{Li}^c e_{Lj} \tilde{e}_{Rk}^* + \bar{e}_{Rk} \nu_{Li} \tilde{e}_{Lj} + \bar{e}_{Rk} e_{Lj} \tilde{\nu}_{Li} - (i \leftrightarrow j)] + \lambda'_{ijk} [\bar{\nu}_{Li}^c d_{Lj} \tilde{d}_{Rk}^* \\ & + \bar{d}_{Rk} \nu_{Li} \tilde{d}_{Rk} + \bar{d}_{Rk} d_{Lj} \tilde{\nu}_{Li} - \bar{e}_{Ri}^c u_{Lj} \tilde{d}_{Rk}^* - \bar{d}_{Rk} e_{Li} \tilde{u}_{Lj} - \bar{d}_{Rk} u_{Lj} \tilde{e}_{Li}] + h.c.. \end{aligned} \quad (46)$$

From Eq. (46), we find that the four-fermion interaction for  $b \rightarrow q\tau\nu$  with the slepton as the intermediate state is given by

$$\mathcal{L}_{RV} = -\frac{\lambda_{3i3}^* \lambda'_{ik3}}{M_{\tilde{e}_{Li}}^2} \bar{b} P_L q \bar{\nu} P_R \tau, \quad (47)$$

where  $k = 1(2)$  for  $q = u(c)$  and  $M_{\tilde{e}_{Li}}^2$  is the slepton mass.

From the interaction in Eq. (47), we get

$$G_P = \frac{\lambda_{3i3}^* \lambda'_{ik3}}{4M_{\tilde{e}_{Li}}^2}, \quad (48)$$

which leads to

$$\Delta_P = \frac{\sqrt{2}}{4G_F V_{qb}} \frac{M_{B_q}^2}{(m_b + m_q)m_\tau} \frac{\lambda_{3i3}^* \lambda'_{ik3}}{M_{\tilde{e}_{Li}}^2} \quad (49)$$

for  $i = 1, 2$ . We therefore obtain the transverse tau polarization of  $B_q^+ \rightarrow \tau^+ \nu_\tau \gamma$  as

$$P_T = -(\sigma_V - \sigma_A) \frac{\sqrt{2}}{4G_F V_{qb}} \frac{M_{B_q}^2}{(m_b + m_q)m_\tau} \frac{\text{Im}(\lambda_{3i3}^* \lambda'_{ik3})}{M_{\tilde{e}_{Li}}^2}. \quad (50)$$

In order to give the bounds on the R-parity violating couplings, we need to examine various processes induced by the FCNC. From the data in Ref. [12] on the leptonic  $\tau$  decays, the bounds on R-parity violating couplings  $\lambda$  are given by [23]

$$\frac{|\lambda_{23k}|}{M} < 4.0 \times 10^{-4} \frac{1}{\text{GeV}}. \quad (51)$$

From the limit on the electron neutrino mass [12], one has [24]

$$\frac{|\lambda_{133}|}{M} < 1.0 \times 10^{-5} \frac{1}{\text{GeV}}. \quad (52)$$

The bounds on  $\lambda'$  can be extracted from the experimental limit on  $K^+ \rightarrow \pi \nu \bar{\nu}$  as studied in Ref. [25]. One finds that

$$\frac{|\lambda'_{ijk}|}{M_{\tilde{d}_{Rk}}} < 1.2 \times 10^{-4} \frac{1}{\text{GeV}}, \quad (53)$$

where  $j = 1, 2$  and  $M_{\tilde{d}_{Rk}} \sim M = 100 \text{ GeV}$  is the sdown-quark mass. From Eqs. (50)-(53), we find

$$P_T(B_{u,c}^+ \rightarrow \tau^+ \nu_\tau \gamma) \leq 0.16, 0.02 \quad (54)$$

where we have used that  $|\sigma_V - \sigma_A| \sim 0.1$ . Similar studies in the various semileptonic B decays have been done in Ref. [26].

## 4 Conclusions

We have studied the transverse tau polarization asymmetries in the decays of  $B_{u,c}^+ \rightarrow \tau^+ \nu_\tau \gamma$  in various CP violation theories. We have demonstrated that the asymmetries are zero in the standard model but they can be large in the non-standard CP violation theories. Explicitly, we have found that  $P_T(B_{u(c)}^+ \rightarrow \tau^+ \nu_\tau \gamma)$  can be large as large as  $3.3 \times 10^{-2}$  ( $3.3 \times 10^{-2}$ ), 0.58 (0.67), and 0.16 (0.02) in the left-right symmetric, three-Higgs doublet, and SUSY models. We note that some of the values could be accessible in the B factories and LHC. For example, experimentally, to observe the tau transverse polarization asymmetries with the efficiency of 4% in detecting the tau lepton in the three-Higgs doublet models at the  $n\sigma$  level, one needs at least  $1.0 \times 10^8$  ( $4.0 \times 10^5$ )  $n^2$   $B_{u(c)}^+$  decays. Clearly, the measurement of such effect is a clean signature of CP violation beyond the standard model.

## ACKNOWLEDGMENTS

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## Figure Captions

Figure 1: Diagrams for “Inner-bremsstrahlung” contributions.

Figure 2: Diagrams for “Structure-dependent” contributions.

Figure 3: Dalitz plots of  $A_0(x, y)$  for (a)  $B^+ \rightarrow \tau^+ \nu_\tau \gamma$  and (b)  $B_c^+ \rightarrow \tau^+ \nu_\tau \gamma$ .

Figure 4: Dalitz plots of  $\sigma_V(x, y)$  for (a)  $B^+ \rightarrow \tau^+ \nu_\tau \gamma$  and (b)  $B_c^+ \rightarrow \tau^+ \nu_\tau \gamma$ .

Figure 5: Dalitz plots of  $\sigma_V(x, y) - \sigma_A(x, y)$  for (a)  $B^+ \rightarrow \tau^+ \nu_\tau \gamma$  and (b)  $B_c^+ \rightarrow \tau^+ \nu_\tau \gamma$ .

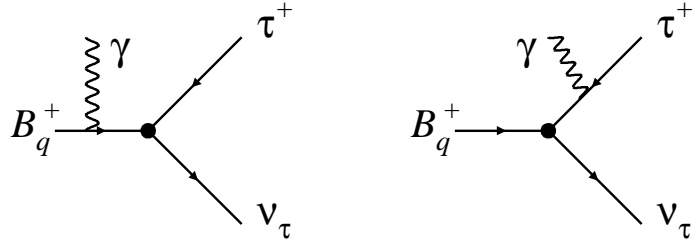


Figure 1: Diagrams for “Inner-bremsstrahlung” contributions.

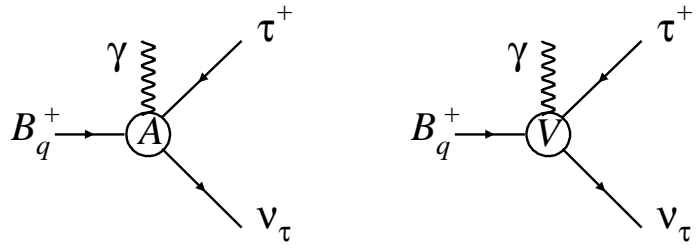


Figure 2: Diagrams for “Structure-dependent” contributions.



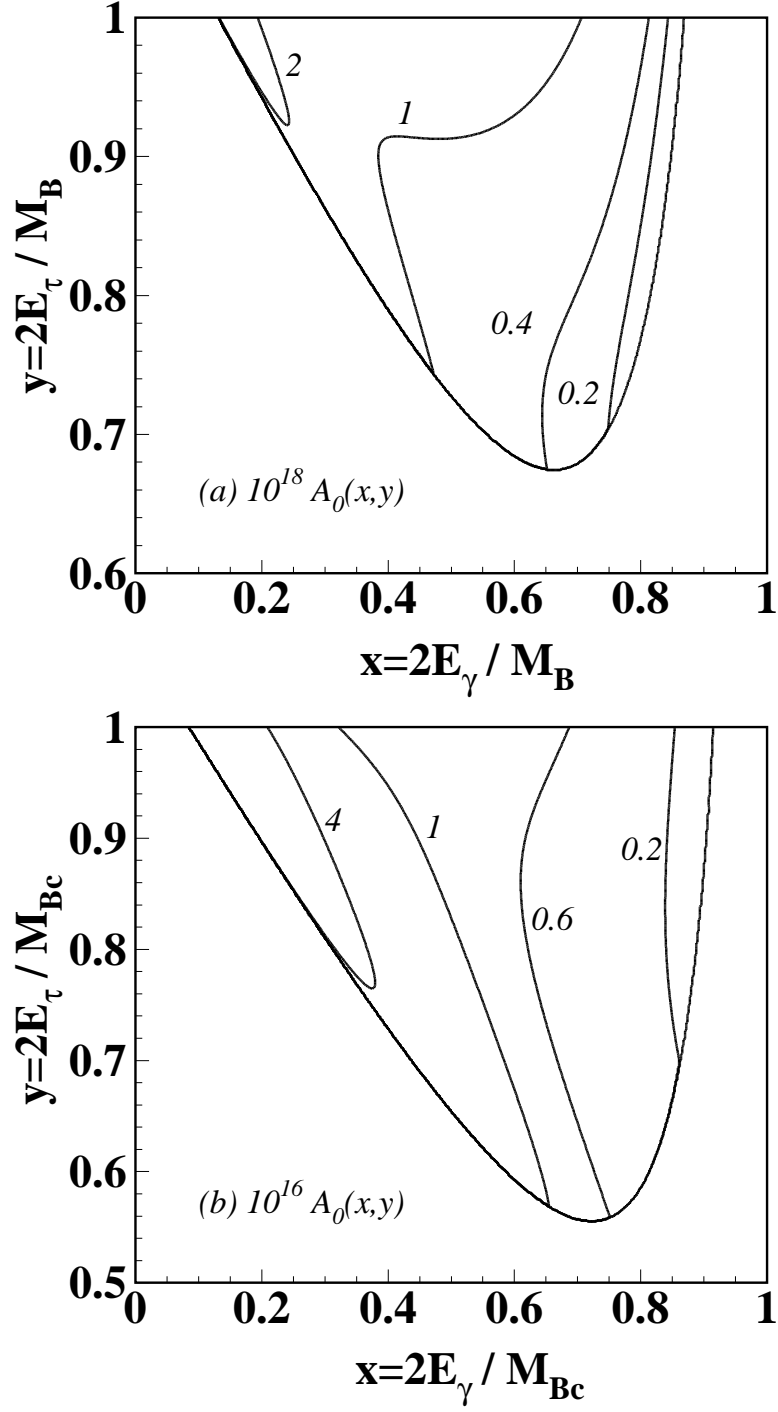


Figure 3: Dalitz plots of  $A_0(x, y)$  for (a)  $B^+ \rightarrow \tau^+ \nu_\tau \gamma$  and (b)  $B_c^+ \rightarrow \tau^+ \nu_\tau \gamma$ .

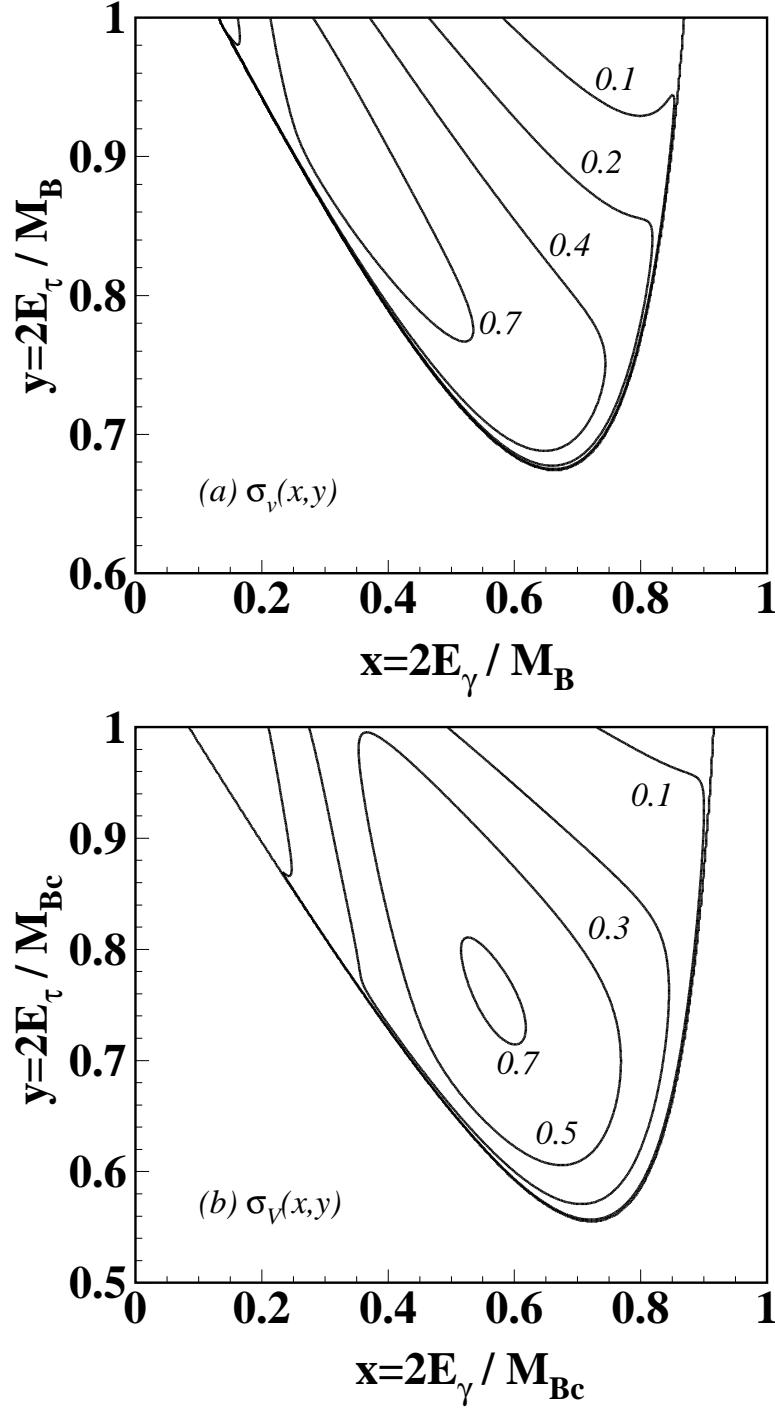


Figure 4: Dalitz plots of  $\sigma_V(x, y)$  for (a)  $B^+ \rightarrow \tau^+ \nu_\tau \gamma$  and (b)  $B_c^+ \rightarrow \tau^+ \nu_\tau \gamma$ .

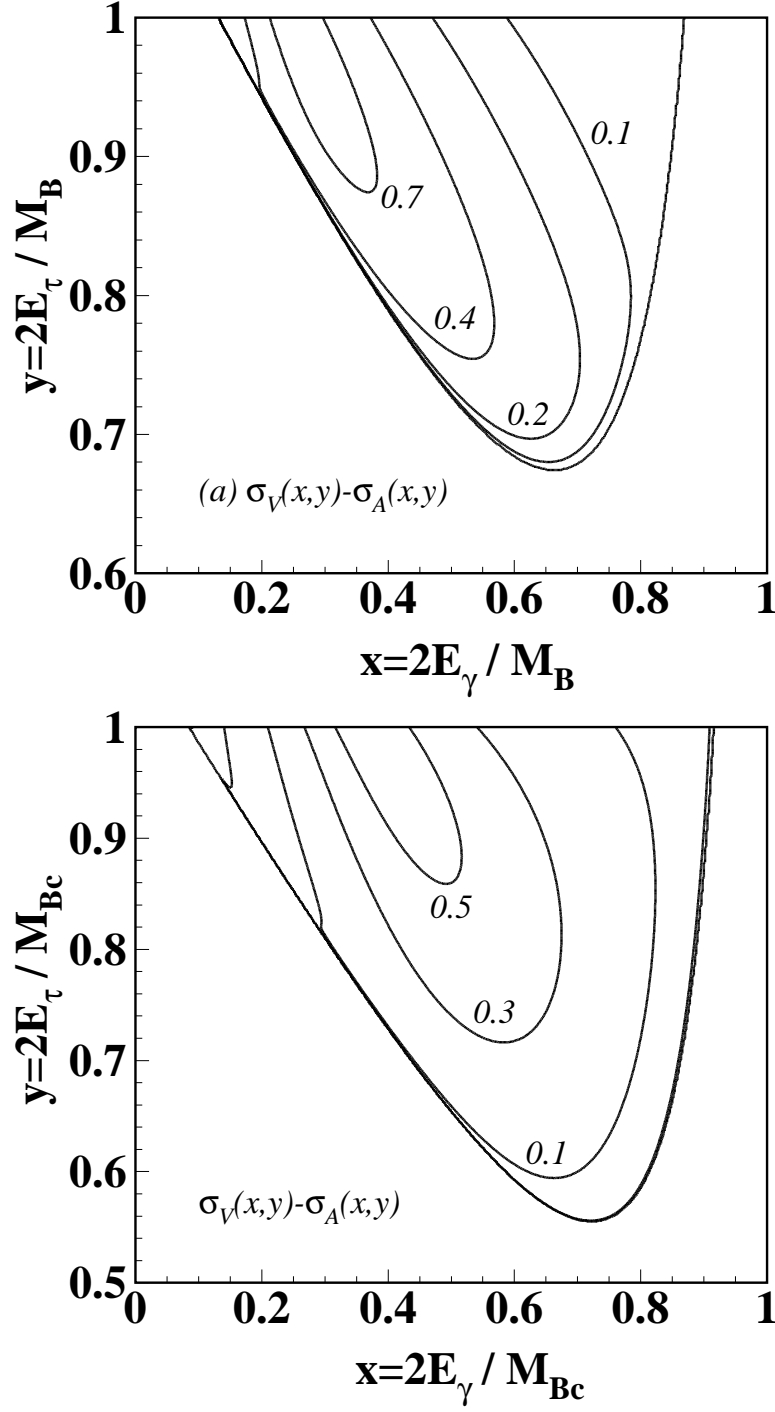


Figure 5: Dalitz plots of  $\sigma_V(x,y) - \sigma_A(x,y)$  for (a)  $B^+ \rightarrow \tau^+ \nu_\tau \gamma$  and (b)  $B_c^+ \rightarrow \tau^+ \nu_\tau \gamma$ .